10. If $f(z)$ be an analytic function in the domain D, and lat z_0 be a point in D. If $f'(z_0) \neq 0$, then $f(z)$ is conformal at z_0.

Section B

Note : Attempt all the questions.

11. (a) State and prove Cauchy-Goursat theorem.

(b) State and prove Taylor's theorem. Also, find Taylor series for the function $f(z) = \dfrac{1+2z^3}{z+z^2}$, valid in a neighbourhood of the point $z = i$.

Or

(a) State and prove Rouche's theorem.

(b) State and prove Poisson's formula. **12**

12. (a) Under the transformation $w = \log z$ find the image region of the annulus $a \leq |z| \leq b$.

Roll No. .... **Exam Code : J-19**

Subject Code—0355

M. Sc. EXAMINATION

(Batch 2011 Onwards)

(First Semester)

MATHEMATICS

MAL-515

Complex Analysis-I

Time : 3 *Hours* *Maximum Marks* : 70

Section A

Note : Attempt any *Seven* questions. **7×5=35**

1. (a) If $f(z)$ is differentiable in a connected open set D and $|f(z)|$ is constant in D, then show that $f(z)$ is a constant in D.

(b) If $f(z)$ is analytic on a simply connected region A then, for any *two* curves γ_1 and γ_2 joining two points z_0 and z_1 then prove that $\int_{y_1} f(z)dz = \int_{\gamma_2} f(z)dz$.

2. Prove that $z = a$ be a zero of order k of an analytic function $f(z)$ iff $f(z) = (z-a)^k \phi(z)$ where $\phi(a) \neq 0, \phi(a) \neq \infty$.

3. (a) If $f(z)$ is continuous in a domain D and iff for every closed contour C in the domain D $\int_C f(z)dz = 0$ then prove that $f(z)$ is analytic within D.

(b) Find $\int_C \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)} dz$, where C is the circle $|z| = 3$.

4. A function which is regular in all finite region of the complex plane, and is bounded, is identically equal to a constant.

5. State and prove maximum modulus principle theorem.

6. If the only singularities of an analytic function, including possibility the 'point at infinity' are poles, then prove that it is a rational function.

7. Classify the singularity of the following at the point mentioned against them :

(a) $f(z) = \frac{1+2z^3}{z+z^2}$ at the point of infinity

(b) $\frac{1}{1-e^z}$ at $z = 2\pi i$.

8. For the transformation $w = z^2$ find images of infinite strips between the lines $x = a$, $x = b$, and $x = -a$, $x = -b$.

9. State and prove Schwarz lemma.

Or

(a) State and prove Laurent theorem. Also find Laurent series expansion, in power of (z + 1), of the function $f(z)=\dfrac{z^2+1}{z(z^2-3z+2)}$ in the region $|z+1|>3$.

(b) If $f(z)$ has an isolated signularity at z_0, then $f(z)$ has a pole at z_0 if and only if $\lim\limits_{z\to z_0} f(z)=\infty$. **11**

(b) State and prove Argument principle theorem.

Or

(a) State and prove Mittag Leffler's expansion theorem.

(b) Use method of contour integration prove that :

$$\int_0^{2\pi}\frac{1}{1+a^2-2a\cos\theta}\,d\theta=\frac{2\pi}{1-a^2},\ 0<a<1.$$

12

13. (a) State and prove sufficient condition for $w=f(z)=u+iv$ to be conformal mapping.

(b) If $f(z)$ is analytic at z_0 with $f(z_0)\neq 0$ and $g(z)$ has a simple zero at z_0 then show that $\text{Res}\left[\dfrac{f(z)}{g(z)};\,z_0\right]=\dfrac{f(z_0)}{g(z_0)}$.