(b) State Dirichlet's test for uniform convergence of series. Using this, show that the series $\sum(-1)^n \frac{x^2+n}{n^2}$ converges uniformly in every bounded interval, but does not converge absolutely for any value of *x*.

12. State and prove inverse function theorem by giving full details. **12**

Or

Prove that the necessary and sufficient condition for a bounded function f to be integrable on $[a, b]$ (i.e. $f \in R_\alpha$) is that to each $\in > 0$, $\exists$ a partition P of $[a, b]$ such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$. Also prove that if $U(P, f, \alpha) - L(P, f, \alpha) < \in$ is true for $P = \{a = x_0, x_1, x_2, ..., x_{i-1}, x_i, ..., x_n = b\}$ and if s_i, t_i are arbitrary points in $[x_{i-1}, x_i]$ then $\sum_{i=1}^{n} \left| f(s_i) - f(t_i) \right| . \Delta\alpha_i < \in$.

Roll No. .... **Exam Code : J-19**

Subject Code—0352

M. Sc. EXAMINATION

(Batch 2011 Onwards)

(First Semester)

MATHEMATICS

MAL-512

Real Analysis

Time : 3 *Hours* *Maximum Marks :* 70

Section A

Note : Attempt any *Seven* questions. **7×5=35**

1. State and prove Cauchy's criterion for uniform convergence of a sequence of functions.

2. Test the uniform convergence of the series $\sum \frac{x}{n\left(1+nx^2\right)}$ $\forall\, x \in \mathbf{R}$, using Weierstrass M-test.

3. If $\{f_n\}$ is a sequence of continuous functions on E and if f_n converges to f uniformly on E, then prove that f is continuous on E. Is the converse true ? Justify your answer.

4. Prove that a linear operator on a finite dimensional vector space X is one-one iff R(T) = X i.e. T is onto.

5. State and prove Young's theorem for the interchange of the order of differentiation.

6. Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the conditions $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$ and $z = x + y$.

7. If f is continuous, then prove that $f \in \mathbf{R}_a$ on $[a, b]$.

8. If f is continuous and α is monotonically increasing on $[a, b]$, then show that :

$$\int_a^b f\, d\alpha + \int_a^b f\, df = f(b)\alpha(b) - f(a)\alpha(a)$$

9. If E_1 and E_2 are measurable sets such that $E_1 \supset E_2$ and $m(E_2) < \infty$, then $m(E_1 - E_2) = m(E_1) - m(E_2)$.

10. Let E be a given set. Then prove that if E is measurable, then given $\in > 0$, $\exists$ open set $0 \supset E$ such that $m^*(0 - E) < \in$.

Section B

Note : Attempt all the questions.

11. State and prove Abel's theorems in both forms (first form and second form). **12**

Or

(a) State and prove Weirstrass M-test for the uniform convergence of a series of functions. Also show that the series $(1-x)^2 + (1-x)^2 x + (1-x)^2 x^2 + ...$ is uniformly convergent on [0, 1].

13. Define Borel σ-algebra and Borel sets. Give an example of each. Also prove that every Borel set in $\mathbf{R}$ is measurable. **11**

Or

Prove that there exists a non-measurable set in the interval [0, 1]. Also show that Cantor's Ternary set has measure zero.

13. Define Borel σ-algebra and Borel sets. Give an example of each. Also prove that every Borel set in $\mathbf{R}$ is measurable. **11**

Or

Prove that there exists a non-measurable set in the interval [0, 1]. Also show that Cantor's Ternary set has measure zero.