**12.** (a) Prove that a family $\beta$ of sets is a base for a topology for the set $X = \cup\{B : B \in \beta\}$ iff for every $B_1, B_2 \in \beta$ and every $x \in B_1 \cap B_2$, $\exists$ a set $B \in \beta$ s.t. $x \in B \subseteq B_1 \cap B_2$. **6**

(b) Show that a top space is compact iff any family of closed sets having FIP has a non-empty intersection. **6**

*Or*

(a) Prove that every sequentially compact top space is countably compact. **6**

(b) Show that every sequentially compact metric space is compact. **6**

**13.** (a) Define first and second axiom space. Also show that every second axioms space is first axiom but converse need not be true. **6**

(b) Prove that every compact Hausdorff space is regular. **5**



**Roll No.** ........................ **Exam Code : J-19**

# Subject Code—0356

## M. Sc. EXAMINATION

(Batch 2011 Onwards)

(Third Semester)

MATHEMATICS

MAL-631

Topology

*Time :* 3 *Hours* *Maximum Marks :* 70

### Section A

**Note** : Attempt any *Seven* questions. **7×5=35**

**1.** State and prove cofinite topology on a non-empty set.

**2.** Define interior of a set. Also show that :

$$\mathring{A} = \{y : y \in A \text{ and } y \notin d(X - A)\}$$

**3.** If $f$ is a mapping of a top space X into another topological space, then prove that $f$ is continuous on X iff

$$f\left(C(E)\right) \subseteq C^* f(E) \;\; \forall E \subseteq X.$$

**4.** If C is a connected subset of a top space (X, T) which has a separation X = A/B, then show that either $C \subseteq A$ or $C \subseteq B$.

**5.** If E is a subset of a subspace (X*, T*) of a top space (X, T), then show that E is T*-compact iff it is T-compact.

**6.** Define locally compact space. Show that every compact top-space is locally compact but converse need not be true.

**7.** Show that a compact subset of a metric space is closed and bounded. Give an example to show that a compact subset of a top space need not be closed.

**8.** Define a separable space. Also show that every second countable space is separable.



**9.** Prove that a top space X is $T_0$-space iff the closures of distinct points are distinct.

**10.** Define product topology and projections $\Pi_x$ and $\Pi_y$ of X × Y on X and Y respectively.

## Section B

**Note** : Attempt all the questions.

**11.** (a) Prove that $\bar{A} = A \cup d(A) \; \forall \; A \subseteq X$. **6**

(b) Define topology in terms of Kurtowski closure operator. **6**

*Or*

(a) Let T be the collection of subset of N consisting of $\phi$ and all subsets of N of the form $A_n = \{n, n+1, n+2, ......\}$, $n \in N$. Show that T is a topology for N. Also find the derived set of {4, 13, 28, 35}. **6**

(b) Show that a top space is connected iff it has no non-empty proper subset which is both open and closed. **6**

*Or*

(a) Prove that a top-space X is completely normal iff every subspace of X is normal. **6**

(b) Show that the property of a space being normal is not a hereditary property. **5**

*Or*

(a) Prove that a top-space X is completely normal iff every subspace of X is normal. **6**

(b) Show that the property of a space being normal is not a hereditary property. **5**